CP Violation Effects on the Measurement of the CKM Angle γ from $B \to DK$

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Inspired by the unexpectedly large difference between the CP violation of D decays into K^+K^- and $\pi^+\pi^-$, we explore the impact on the extraction of γ via the $B \to DK$ process with the D meson reconstructed in the $K^+K^-, \pi^+\pi^-$ final state. We show that the extracted results for γ can be shifted by $\mathcal{O}(A_{CP}/r_B^K)$, where A_{CP} is the direct CP asymmetry in D decays and r_B^K is the ratio of the decay amplitudes of $B^- \to \bar{D}^0 K^-$ and $B^- \to D^0 K^-$. Using the recent data on CP asymmetry, we demonstrate the correction to physical observables in $B \to DK$ can reach 6%, which corresponds to the shift of γ by roughly 5°. The remanent corrections depend on the strong phase of the D decays, but are less than 0.5°. With the increasing precision in the γ determination on the LHCb experiment and Super B factories, the inclusion of CP violation of D decays will therefore soon become important.

PACS numbers: 13.25.Hw,12.15.Hh

Introduction.—The fundamental role in the standard model (SM) description of CP violation is played by the unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The constraints on this matrix can be represented as triangles, the lengths of whose sides are the moduli of CKM matrix element products, while the angles represent relative phases. Among the three angles (α, β, γ) of the (bd) unitarity triangle, satisfying the constraint $\alpha + \beta + \gamma = 180°$, γ is least well known, with a precision of roughly 10°. This is one of the main sources of current uncertainties on the apex of the unitary triangle [1].

In contrast with the α, β angles whose determination is often challenged by the loop penguin pollutions on theoretical side, one of the most intriguing properties of the angle γ is that it can be measured in a way independent of hadronic uncertainty [2–4]. That makes use of the tree-dominated processes $B \to DK$. With a large amount of data accumulated in the future, the LHCb will be able to diminish the errors in γ to about 4° from the tree-dominated processes $B \to DK$ [5], while on the SuperB factories the error is further reduced to 2° [6, 7]. Hopefully with these results, one may be able to authenticate the unitarity of CKM matrix, and thus the relevance of New Physics for the phenomena of flavor physics or not.

Recently one of the most exciting measurements by LHCb collaboration [8], confirmed by CDF [9] and Belle [10] collaborations, is the CP violation (CPV) in charm sector. These three collaborations have found nonzero difference of CP asymmetries (CPAs) which are much larger than the SM expectation. The averaged results are [1]

$$\begin{aligned}\Delta a_{CP} &= a_{CP}(K^+K^-) - a_{CP}(\pi^-\pi^+) \\ &= (-0.74 \pm 0.15)\%.\end{aligned} \tag{1}$$

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Here $a_{CP}(f)$ is the time integrated CP asymmetry for D decaying into a CP eigenstate:

$$a_{CP}(f) = \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to f)}. \tag{2}$$

The dominant contribution to Δa_{CP} in Eq. (1) is from the direct CP violation of D^0 decays [1]:

$$\Delta A_{CP}^{\rm dir} = (-0.678 \pm 0.147)\%, \tag{3}$$

which is about 4.6σ away from 0.

On the one hand it is desirable to look for more precise measurement to establish the large CPA and identify possible future experimental tests able to distinguish the standard model vs new physics interpretations. On the other hand, it is of great importance to investigate the impact of a_{CP} on the known phenomena. In this work, we are interested in the effects of the nonzero CPA on the determination of the γ angle, in particular in the method to use the $B \to DK$ with D decays into CP eigenstate K^+K^- or $\pi^+\pi^-$, the so-called the GLW method [2]. We will demonstrate that the CPA effects can shift the γ by a few degrees. These corrections are larger than or comparable with the future experimental precision and thus must be included in the future measurement.

γ measurement via $B \to D_{CP} K$.—Let us start with a review of the approach with the negligence of the CP violation effects. The GLW method uses the fact that the six decay amplitudes of $B^\pm \to (D^0, \bar{D}^0, D^0_{CP})K^\pm$ form two triangles in the complex plane, graphically representing the identities

$$\begin{aligned}\sqrt{2}A(B^+ \to D^0_\pm K^+) &= A(B^+ \to D^0 K^+) \\ &\quad \pm A(B^+ \to \bar{D}^0 K^+), \\ \sqrt{2}A(B^- \to D^0_\pm K^-) &= A(B^- \to D^0 K^-) \\ &\quad \pm A(B^- \to \bar{D}^0 K^-),\end{aligned} \tag{4}$$

where the convention $CP|D^0\rangle = |\bar{D}^0\rangle$ has been adopted and $D^0_+ (D^0_-)$ is the CP even (odd) eigenstate. The

K^+K^- and $\pi^+\pi^-$ final states will mainly project out the D^0_+, which will be considered in the following. Measurements of the six decay rates will completely determine the sides and apexes of the two triangles, in particular the relative phase between $A(B^- \to \bar{D}^0 K^-)$ and $A(B^+ \to D^0 K^+)$ is 2γ. Since the identities in Eq. (4) holds irrespective of the strong phase in the decay, this method is free of hadronic uncertainties and is believed to be theoretically clean.

Different with the loop-induced processes, tree-dominated processes are unlikely affected by the new physics degrees of freedoms and thus the measurement of γ provides a benchmark of extraction of the CKM parameters.

The shape of the two triangles is governed by two quantities

$$r_B^K \equiv \left|A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-)\right|,$$
$$\delta_B^K \equiv arg\left[e^{i\gamma}A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-)\right],$$

with the world averages for these parameters [11]

$$r_B^K = 0.107 \pm 0.010, \quad \delta_B^K = (112^{+12}_{-13})^\circ. \tag{5}$$

The smallness of r_B^K implies the mild sensitivity to γ and introduces experimental difficulty. Thus additional methods using processes in which the D^0 and $\bar{D}^0$ meson can be accessed in the same final states are proposed [3, 4].

CPA effects.—We now study the effects of CP violation but neglect the small D mixing effects [12]. In particular, we ask how large is the error introduced in the extracted value when the analysis is done assuming no CPV.

The D meson decay amplitudes can be generically decomposed as

$$\begin{aligned} A(D^0 \to f) &= T_D^f(1 + r_D^f e^{-i\gamma + i\delta_D^f}), \\ A(\bar{D}^0 \to f) &= T_D^f(1 + r_D^f e^{i\gamma + i\delta_D^f}), \end{aligned} \tag{6}$$

where r_D^f is the ratio of the penguin and tree amplitudes in $D \to f$ decays, with $f = K^+K^-, \pi^+\pi^-$. γ and δ_D^f is the weak phase difference and strong phase difference respectively. The direct CP asymmetry is predicted as

$$A_{CP}^{dir}(D^0 \to f) = \frac{2r_D^f \sin\gamma \sin\delta_D^f}{1 + (r_D^f)^2 + 2r_D^f \cos\gamma\cos\delta_D^f}. \tag{7}$$

The data on A_{CP} in Eq. (3) indicates $|r_D^f| \sim \mathcal{O}(10^{-3})$.

Including the CP violating amplitudes of the D decays, we arrive at

$$\begin{aligned} \sqrt{2}A(B^- \to D^0_+(\to f)K^-) &= A(B^- \to D^0 K^-)T_D^f\left[(1 + r_D^f e^{-i\gamma + i\delta_D^f}) + r_B^K e^{-i\gamma + i\delta_B^K}(1 + r_D^f e^{i\gamma + i\delta_D^f})\right], \\ \sqrt{2}A(B^+ \to D^0_+(\to f)K^+) &= A(B^+ \to D^0 K^+)T_D^f\left[r_B^K e^{i\gamma + i\delta_B^K}(1 + r_D^f e^{-i\gamma + i\delta_D^f}) + (1 + r_D^f e^{i\gamma + i\delta_D^f})\right], \end{aligned} \tag{8}$$

which apparently will spoil the identities in Eq. (4) since it is not possible to simultaneously eliminate the nontrivial dependence on r_D^f and δ_D^f in the two amplitudes.

The physical observables to be experimentally measured and used to extract the CKM angle γ are given as

$$\begin{aligned} R_+^K &= 2\frac{\mathcal{B}(B^- \to D^0_+ K^-) + \mathcal{B}(B^+ \to D^0_+ K^+)}{\mathcal{B}(B^- \to D^0 K^-) + \mathcal{B}(B^+ \to \bar{D}^0 K^+)} \\ &= 1 + (r_B^K)^2 \\ &\quad + \frac{2r_B^K \cos\delta_B[(1 + (r_D^f)^2)\cos\gamma + 2r_D^f\cos\delta_D^f]}{1 + (r_D^f)^2 + 2r_D^f\cos\gamma\cos\delta_D^f}, \\ &\equiv 1 + (r_B^K)^2 + 2r_B^K\cos\delta_B^K\cos\gamma_{eff}, \end{aligned} \tag{9}$$

$$\begin{aligned} A_+^K &= \frac{\mathcal{B}(B^- \to D^0_+ K^-) - \mathcal{B}(B^+ \to D^0_+ K^+)}{\mathcal{B}(B^- \to D^0_+ K^-) + \mathcal{B}(B^+ \to D^0_+ K^+)} \\ &= \frac{1}{R_+^K}\Bigg[(1 - (r_B^K)^2)A_{CP}^{dir}(D^0 \to f) \\ &\quad + \frac{2r_B^K(1 + (r_D^f)^2)\sin\delta_B^K\sin\gamma}{1 + (r_D^f)^2 + 2r_D^f\cos\delta_D^f\cos\gamma}\Bigg] \\ &\equiv 2r_B^K\sin\delta_B^K\sin\gamma_{eff}/R_+^K, \end{aligned} \tag{10}$$

where the last lines in the above equations correspond to the expressions with no CPV effects. In the above expressions, we have substituted the CP averaged branching ratio of $D \to f$ in the processes involving D^0_+. These two equations explicitly show the CPA effects on the experimental observables and are one of the main findings of this work.

Results.—An interesting observation is that the A_+^K in Eq. (10) receives new contributions proportional to the direct CPA in D decays. Neglecting terms suppressed by $\mathcal{O}(r_D^f)$, the dominant correction to $\sin\gamma$ is proportional to $A_{CP}^{dir}(D^0 \to f)/(2r_B^K\sin\delta_B)$. One consequence is that the

value for γ obtained from K^+K^- final state and the one from $\pi^+\pi^-$ final states will differ roughly by 5°. These effects can still be incorporated without any hadronic uncertainty once the data on the direct CPA is available.

For illustration, we consider three general pattens for the CPV: (i) $A_D^{K^+K^-} = -A_D^{\pi^+\pi^-} = \Delta A_{CP}/2$; (ii) $A_D^{K^+K^-} = \Delta A_{CP}$ and $A_D^{\pi^+\pi^-} = 0$; (iii) $A_D^{K^+K^-} = 0$ and $A_D^{\pi^+\pi^-} = -\Delta A_{CP}$. In the first patten, considering the dominant corrections in the expressions for A_+^K in Eq. (10) to extract the γ, we find that the difference $\Delta\gamma = \gamma_{eff} - \gamma$ is roughly -2.5° in the K^+K^- final state, while it is 2.5° in the $\pi^+\pi^-$ final state. In the second patten of CPV, the modification in K^+K^- is roughly -5° while the extracted value for γ from the $\pi^+\pi^-$ channel is almost unchanged. The last patten is analogous to the second one except that the shift in $\pi^+\pi^-$ is 5° while the change in K^+K^- mode is negligible. The errors from the current LHCb measurements on A_+^K from the K^+K^- and $\pi^+\pi^-$ final states are too large to give any conclusive result [13]

$$A_+^K(K^+K^-) = 0.148 \pm 0.037 \pm 0.010,$$
$$A_+^K(\pi^+\pi^-) = 0.135 \pm 0.066 \pm 0.010. \quad (11)$$

Apart from the corrections proportional to the direct CPA, there are still more suppressed terms depending on the strong phase δ_D^f as shown in Eq. (9) and Eq. (10). The suppressed CP violation effect in the R_+^K is shown in Fig. 1. The solid (black), dashed(blue), and dotted (red) lines correspond to r_D^f=0.002, 0.004 and 0.006 respectively. The shadowed region is the data on the CPA for $D^0 \to K^+K^-$ in the first patten for CPV: $A_{CP}^{K^+K^-} = (-0.34 \pm 0.07)\%$. From the figure we can see the difference between the results for γ with and without the CPV effects is up to 0.5°, comparable with the experimental accuracy (0.8°) in γ after the LHCb upgrade [5].

Discussions.—The dominant corrections proportional to the $A_{CP}^{\rm dir}$ can be incorporated without any hadronic uncertainty, once the data on the direct CPA is available. However using only $B \to DK \to (K^+K^-, \pi^+\pi^-)$, it is unlikely to eliminate the dependence on the strong phase for the rest terms in Eq. (9) and Eq. (10). It might be possible when results from the charm factory are available and more relevant channels like $B \to DK^*$ are used.

Including the new Belle measurement, the combined result for ΔA_{CP} is about 4.6σ away from 0 as shown in Eq. (3). Still to date the nonzero ΔA_{CP} is not well-established. With the LHCb result based on a small fraction of data recorded so far, significant improvements are expected in the near future. Moreover to explicitly explore the CPV effects on the extraction of γ, one also has to wait for our experimental colleagues to measure the CPV for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ individually. The present results for the individual CP asymmetries from CDF [15] and Belle [10] collaborations are consistent with zero and are not conclusive.

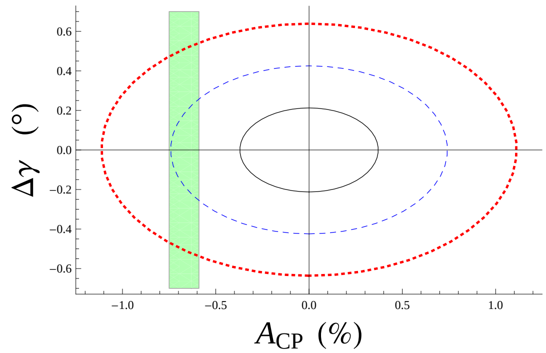

FIG. 1: Effect of CP violation on the extraction of γ via the R_+^K. The sold (black), dashed(blue), and dotted (red) lines correspond to r_D^f=0.002, 0.004 and 0.006 respectively. The shadowed region is the CPA for $D^0 \to K^+K^-$ from the experimental data: $A_{CP}^{K^+K^-} = (-0.34 \pm 0.07)\%$, where we have assumed the U-spin symmetry for the CP asymmetry $A_{CP}^{K^+K^-} = -A_{CP}^{\pi^+\pi^-} = \Delta A_{CP}/2$.

On the experimental side, one may instead use the averaged results from the K^+K^- and $\pi^+\pi^-$ final states. Such procedure will indeed smear most of the CPV effects in the first patten for CP violations, but not in the other two patterns.

The analysis is similar in channels like $B \to DK^*$ in which the D meson is reconstructed in $K^+K^-, \pi^+\pi^-$ final states. In the processes of $B \to DK_{0,2}^*$ [14], which are likely to have a larger ratio $r_B^{K_0^*}$ due to comparable size of the color-allowed and color-suppressed amplitudes, the CP violating corrections, proportional to $A_{CP}^{\rm dir}/r_B^{K^{*0}}$, will be somewhat smaller.

At last, it is worthwhile to stress that the CP asymmetry may not affect the ADS method and the Dalitz plot method. In the former, the doubly-Cabibbo suppressed (DCS) D decays is used to increase the sensitivity while in the Dalitz plot method, the three-body $D \to K_S\pi^+\pi^-$ and $D \to K_S K^+K^-$ decays are used to reconstruct the D_+. The involved Cabibbo allowed and DCS decay modes are typically having very small CPAs and thus the extraction of γ is almost unaffected when neglecting the CPA. Moreover one of the greatest advantages of the Dalitz plot method is that the CPA effects and the small DCS contributions can be explicitly incorporated when enough data is accumulated.

Conclusions.—After roughly 40 years since the proposal, the CKM mechanism continues to provide a consistent description of almost all available data on the flavor observables and CP violation with an impressive accuracy. This great success implies that the New Physics effects should be small, and renders the precision predictions for the involved quantities particularly important. Thus the precise determination of the CKM angles is of great importance, for which one of the most desirable

objectives in the ongoing and forthcoming experiments is to reduce the errors in the involved entries of the unitary triangle.

What has been explored in this work is to study the CPV effects in the GLW method for the extraction of CKM angle γ and showed that the CPV corrections are of the order A_{CP}/r_B^K. If CP violation in D decays is ignored in the extraction of γ, we have found the corrections can reach 6% and the resulting shift of γ is about 5°, larger than or comparable with the precision to be achieved on the future LHCb (4°) and SuperB (2°) experiments. With the increasing precision in the γ determination by these experiments the inclusion of CPV of D decays will therefore soon become mandatory.

Acknowledgement.—The author is grateful to Chuan-Hung Chen, Malcolm John and Yue-Hong Xie for useful discussions. He also thanks Institute of High Energy Physics and Tianjin University for their hospitalities during his visit when part of this work was done. This work is supported in part by the DFG and the NSFC through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD".

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